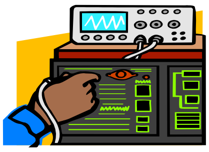
NOTRE DAME UNIVERSITY

FACULTY OF ENGINEERING

Department of Electrical, Computer & Communication Engineering

EEN 202



Circuits Analysis II

Solutions* for HW2

Chapter 8

AC Steady-State Analysis

* Problem Solutions Extracted from the Solution Manual of:
"Basic Engineering Circuit Analysis", by J.D. Irwin and R. M. Nelms, 9th Edition, Wiley, 2008

Instructor: G. Hassoun

8.58 In the network in Fig. P8.58 $\mathbf{v}_o$ is known to be $4\angle 45°$. Find **Z**.

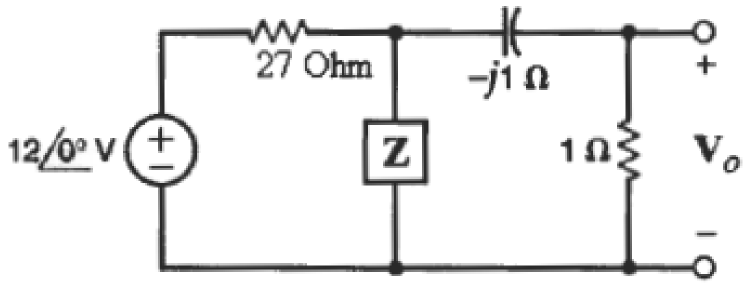
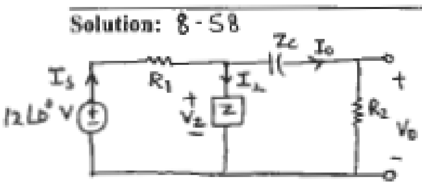


Figure P8.58

(a) Find the real part of **Z**.
(b) Find the imaginary part of **Z**.

Solution: 8-58



$R_1 = 27\,\Omega$
$R_2 = 1\,\Omega$
$Z_c = -j1\,\Omega$

$$I_o = \frac{V_o}{R_2} = 4\angle 45° \text{ A}$$

$$V_z = I_o(R_2 + Z_c) = 5.656\angle 0° \text{ V}$$

$$\text{KVL}: \quad -12\angle 0° + I_s R_1 + V_z = 0$$

$$I_s = \frac{12\angle 0° - V_z}{R_1} = 0.2349 \text{ A}$$

$$I_z = I_s - I_o = -2.5935 - j2.8284 \text{ A}$$

$$Z = \frac{V_z}{I_z} = -0.996 + j1.086\,\Omega$$

$$\boxed{Re(z) = -0.996\,\Omega}$$

$$\boxed{Im(z) = 1.09\,\Omega}$$

8.63 Using nodal analysis, find $\mathbf{I}_o$ in the circuit in Fig. P8.63.

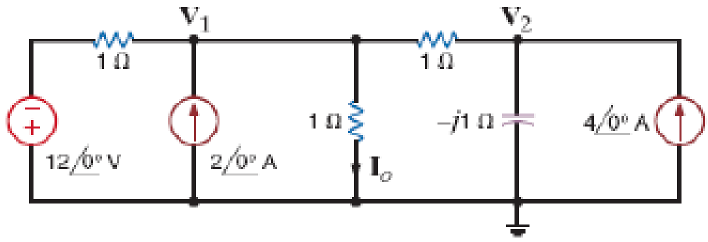
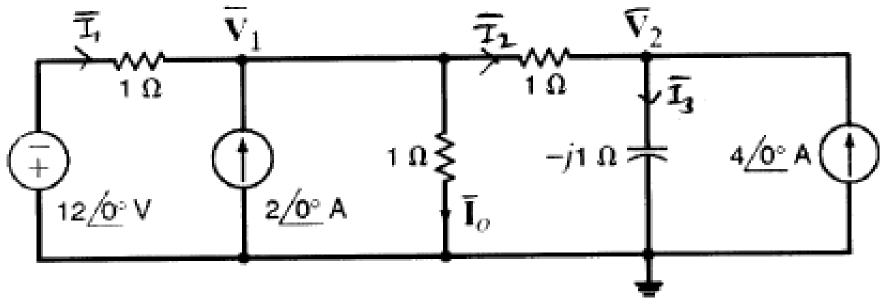


Figure P8.63

SOLUTION:



KCL at ① : $\bar{I}_1 + 2\angle 0° = \bar{I}_o + \bar{I}_2$

$$\frac{-12\angle 0° - \bar{V}_1}{1} + 2\angle 0° = \frac{\bar{V}_1}{1} + \frac{\bar{V}_1 - \bar{V}_2}{1}$$

$$-12\angle 0° - \bar{V}_1 + 2\angle 0° = \bar{V}_1 + \bar{V}_1 - \bar{V}_2$$

$$3\bar{V}_1 - \bar{V}_2 = 10\angle 180°$$

KCL at ② : $\bar{I}_2 + 4\angle 0° = \bar{I}_3$

$$\frac{\bar{V}_1 - \bar{V}_2}{1} + 4\angle 0° = \frac{\bar{V}_2}{-j1}$$

$$-j1(\bar{V}_1 - \bar{V}_2) - j1(4\angle 0°) = \bar{V}_2$$

$$-j1\,\bar{V}_1 + (-1+j1)\,\bar{V}_2 = 4\angle 90°$$

$$3\bar{V}_1 - \bar{V}_2 = 10\angle 180^\circ$$

$$-j1\bar{V}_1 + (-1+j1)\bar{V}_2 = 4\angle 90^\circ$$

$$\bar{V}_1 = 3{\cdot}23\angle -177{\cdot}3^\circ \text{ V}$$

$$\bar{V}_2 = 0{\cdot}55\ \angle -56{\cdot}3^\circ \text{V}$$

$$\bar{I}_0 = \frac{\bar{V}_1}{1}$$

$$\bar{I}_0 = \frac{3{\cdot}23\angle -177{\cdot}3^\circ}{1}$$

$$\bar{I}_0 = 3{\cdot}23\ \angle -177{\cdot}3^\circ \text{A}$$

8.73 Find the voltage across the inductor in the circuit shown in Fig. P8.73 using nodal analysis.

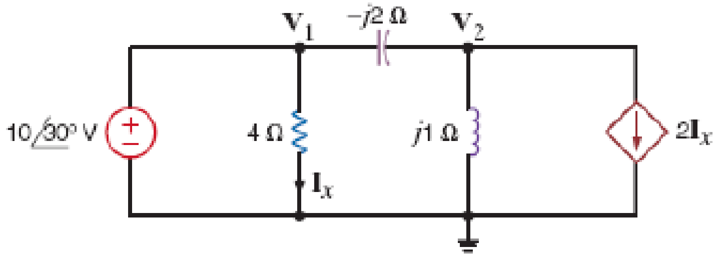
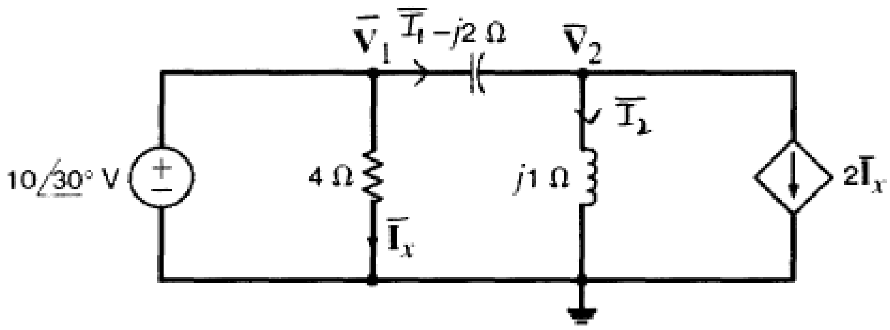


Figure P8.73

SOLUTION:



$$\bar{I}_x = \frac{10\angle 30°}{4} = 2.5\angle 30° \text{ A}$$

KCL at ② : $\bar{I}_1 = \bar{I}_2 + 2\bar{I}_x$

$$\frac{10\angle 30° - \bar{V}_2}{-j2} = \frac{\bar{V}_2}{j1} + 2(2.5\angle 30°)$$

$$10\angle 30° - \bar{V}_2 = -2\bar{V}_2 + 2(2.6\angle 30°)(-j2)$$

$$\bar{V}_2 = 14.14\angle -105° \text{V}$$

8.75 Use mesh analysis to find v_o in the circuit shown in Fig. P8.75.

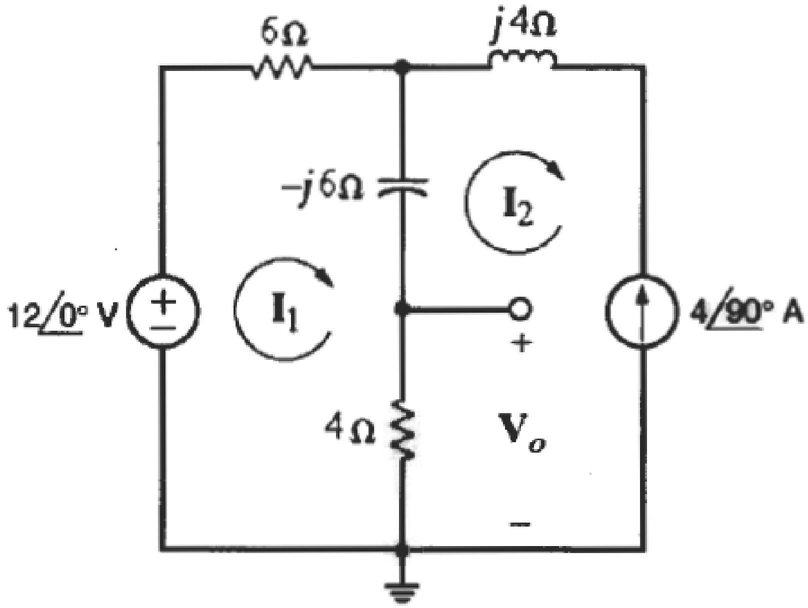


Figure P8.75

Enter **(a)** the amplitude and **(b)** the angle of phase. The angle of phase must be in the interval (−180°,180°).

Solution: 8·75

KVL @ I_1: $-12\angle 0^\circ + I_1(6-j6+4) - I_2(4-j6) = 0$

$$I_1(10-j6) + I_2(-4+j6) = 12$$

$$I_2 = -4\angle 90^\circ = -j4 \text{ A}$$

$$\begin{bmatrix} 10-j6 & -4+j6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -j4 \end{bmatrix}$$

$I_1 = -0.176 - j\,1.7059$ A

$I_2 = -j4$ A

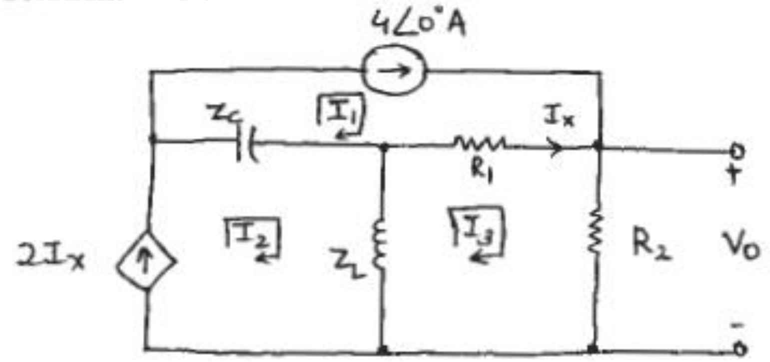
$V_o = (I_1 - I_2)4 = 4(-0.1765 - j\,2.2941)$

$= -0.706 - j\,9.1764$ V

(a) $\boxed{\text{Amplitude of } V_o = 9.20 \text{ V}}$

(b) $\boxed{\text{Angle of phase of } V_o = 85.6^\circ}$

Solution: 8-84



$R_1 = 1\Omega$, $R_2 = 7\Omega$, $Z_C = -j1\Omega$, $Z_L = j1\Omega$

$I_1 = 4\angle 0^\circ$ A — ①

$I_x = I_3 - I_1$

$I_2 = 2I_x$

$\Rightarrow I_2 = 2I_3 - 2I_1 \Rightarrow 2I_1 + I_2 - 2I_3 = 0$ — ②

8.84 Find $\mathbf{v}_o$ in the network in Fig. P8.84.

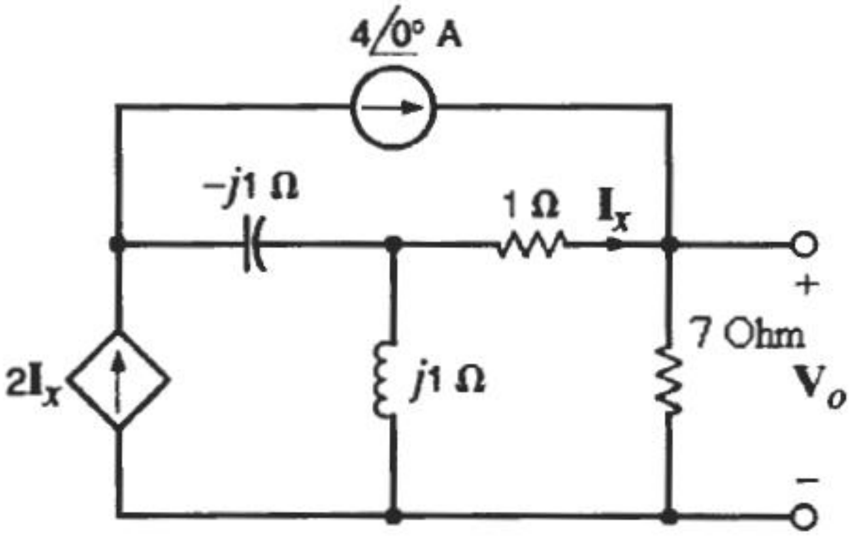


Figure P8.84

(a) Find the real part of $\mathbf{v}_o$.
(b) Find the imaginary part of $\mathbf{v}_o$.

KVL @ I_3 : $(I_3 - I_2)(j1) + (I_3 - I_1)1 + 7I_3 = 0$

$$-I_1 - jI_2 + I_3(8+j) = 0 \qquad — ③$$

From equations ①, ② and ③, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ -1 & -j & 8+j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$V_0 = 7I_3 = 4.31 - j6.46$ V

(a) $\boxed{Re(V_0) = 4.31 \text{ V}}$

(b) $\boxed{Im(V_0) = -6.46 \text{ V}}$

8.88 Using superposition, find v_o in the circuit in Fig. P8.88.

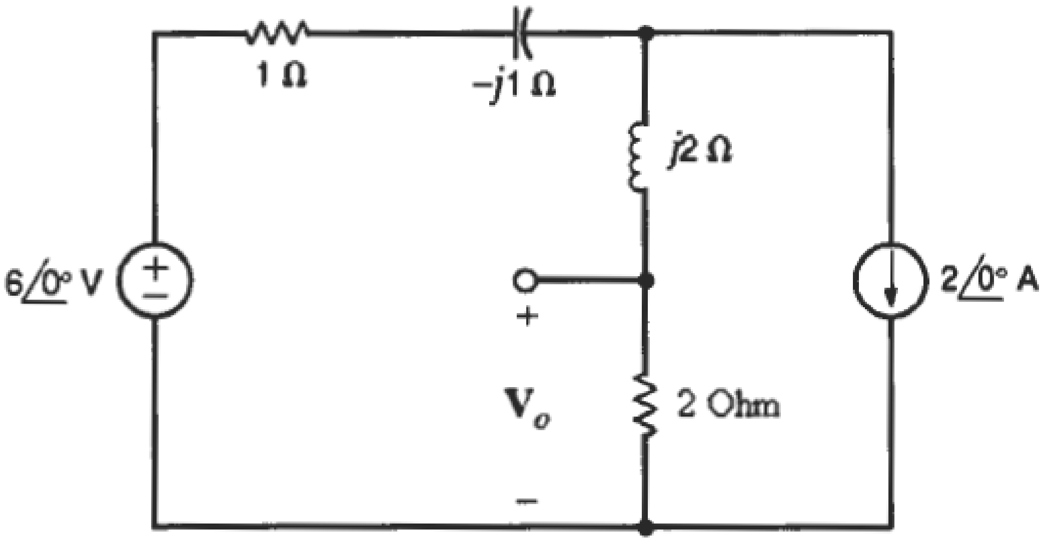
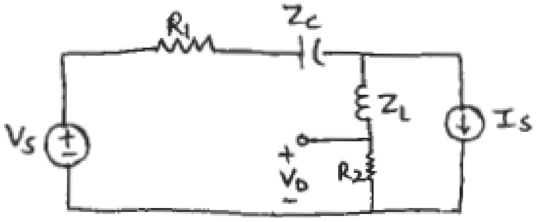


Figure P8.88

(a) Find the real part of v_o.
(b) Find the imaginary part of v_o.

Solution: 8.88

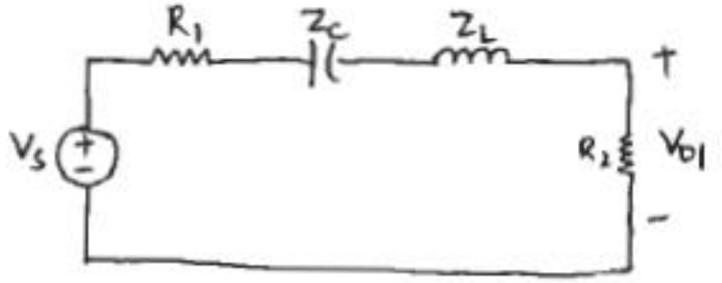


$R_1 = 1\Omega$, $R_2 = 2\Omega$, $Z_C = -j1\Omega$,

$Z_L = j2\Omega$

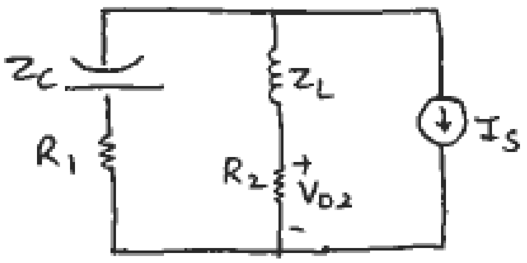
$V_S = 6\angle 0°$ V

$I_S = 2\angle 0°$ A



$$V_{01} = V_s \frac{R_2}{R_1 + R_2 + Z_C + Z_L}$$

$$V_{01} = 3.6 - j\,1.2 \text{ V}$$



$$V_{02} = -I_S \cdot \frac{R_1 + Z_C}{R_1 + R_2 + Z_C + Z_L} \cdot R_2$$

$$V_{02} = -0.8 + j\,1.6 \text{ V}$$

$$V_0 = V_{01} + V_{02} = 2.8 + j\,0.4 \text{ V}$$

(a) $\boxed{\mathrm{Re}(V_0) = 2.8 \text{ V}}$

(b) $\boxed{\mathrm{Im}(V_0) = 0.4 \text{ V}}$

8.93 Use source exchange to determine v_o in the network in Fig. P8.93.

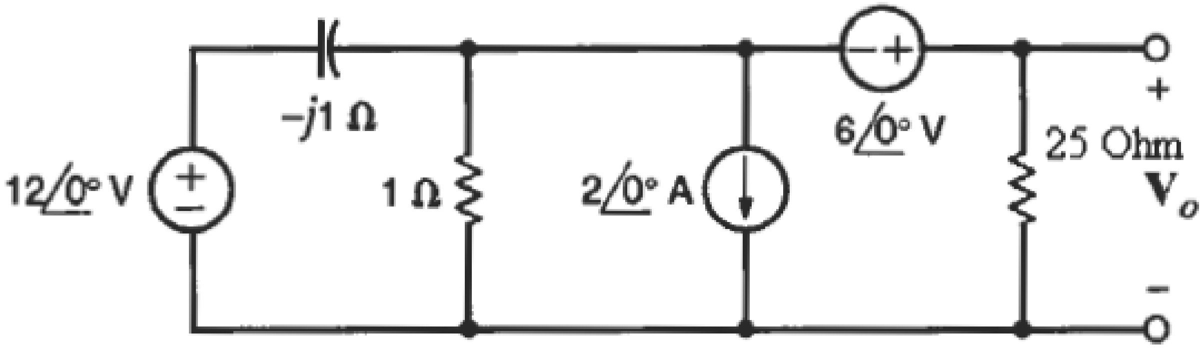
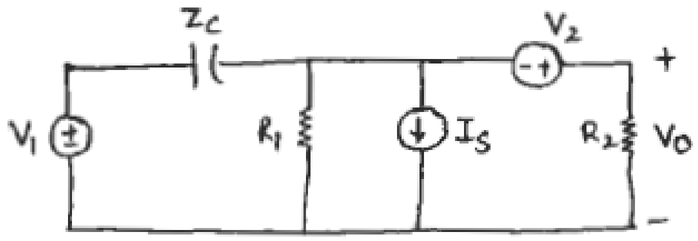


Figure P8.93

(a) Find the real part of v_o.
(b) Find the imaginary part of v_o.

Solution:



$V_1 = 12\angle 0°$ V

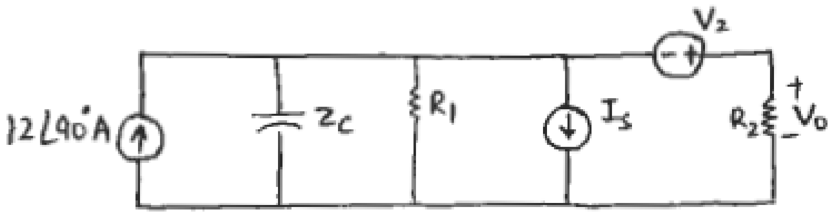
$V_2 = 6\angle 0°$ V

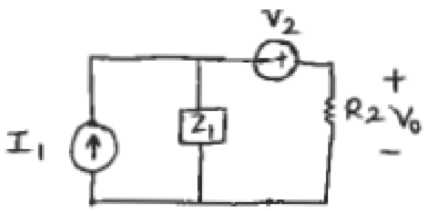
$I_S = 2\angle 0°$ A

$R_1 = 1\,\Omega$

$R_2 = 25\,\Omega$

$Z_C = -j1\,\Omega$

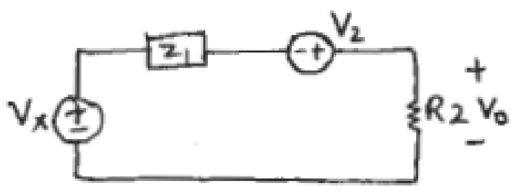




$I_1 = -2 + j\,12$ A

$Z_1 = Z_C \| R_1 = (-j1) \| 1 = \dfrac{-j1}{1-j1}$

$Z_1 = 0.5 - j\,0.5$



$V_x = I_1 Z_1$
$= 5 + j7$ V

$V_0 = (V_x + V_2)\dfrac{R_2}{R_2 + Z_1} = 10.6 + j\,7.07$ V

$\boxed{Re(V_0) = 10.6\ V}$ $\boxed{Im(V_0) = 7.07\ V}$

8.104 Find v_o in the network in Fig. P8.104 using Thévenin's theorem.

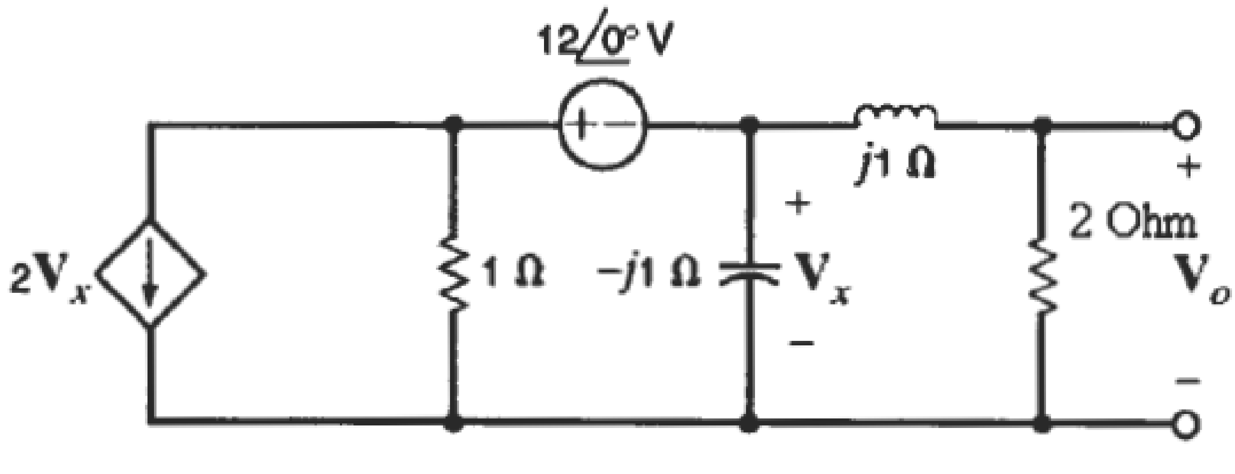
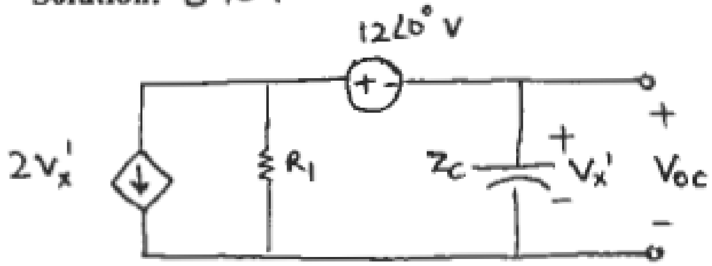


Figure P8.104

(a) Find the real part of v_o.
(b) Find the imaginary part of v_o.

Solution: 8.104

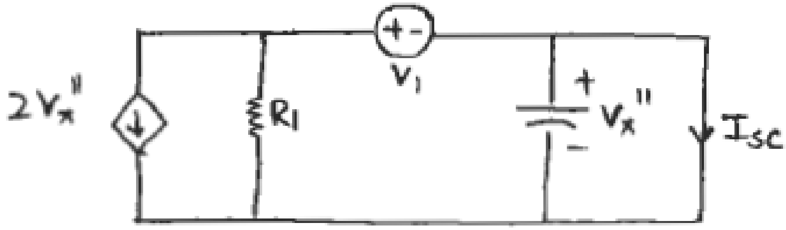


$R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$,
$Z_C = -j1\,\Omega$, $Z_L = j1\,\Omega$

$V_x' = V_{oc}$

$$2V_{oc} + \frac{V_{oc} + V_1}{R_1} + \frac{V_{oc}}{Z_C} = 0$$

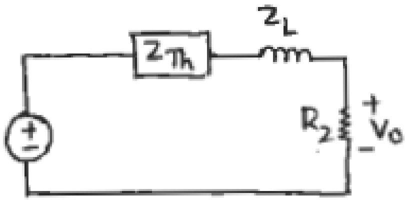
$V_{oc} = -3.6 + j\,1.2\text{ V}$



$V_x'' = 0$

$$\frac{V_1}{R_1} + I_{sc} = 0$$

$I_{sc} = -12\angle 0^\circ\text{ A}$



$$Z_{Th} = \frac{V_{oc}}{I_{sc}} = 0.3 - j\,0.1\ \Omega$$

$$V_o = V_{oc} \cdot \frac{R_2}{R_2 + Z_L + Z_{Th}}$$

$$= -2.36 + 1.97\ V$$

(a) $\boxed{Re(V_o) = -2.36\ V}$

(b) $\boxed{Im(V_o) = 1.97\ V}$

8.105 Find the Thévenin's equivalent for the network in Fig. P8.105 at terminals A-B.

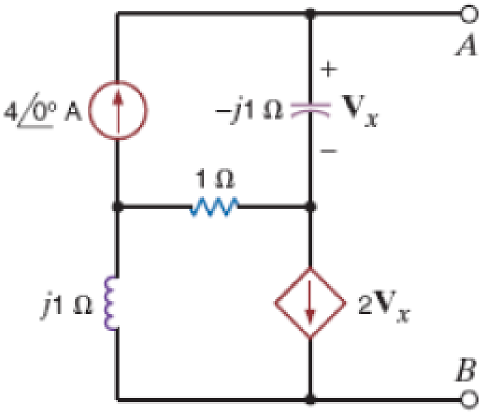
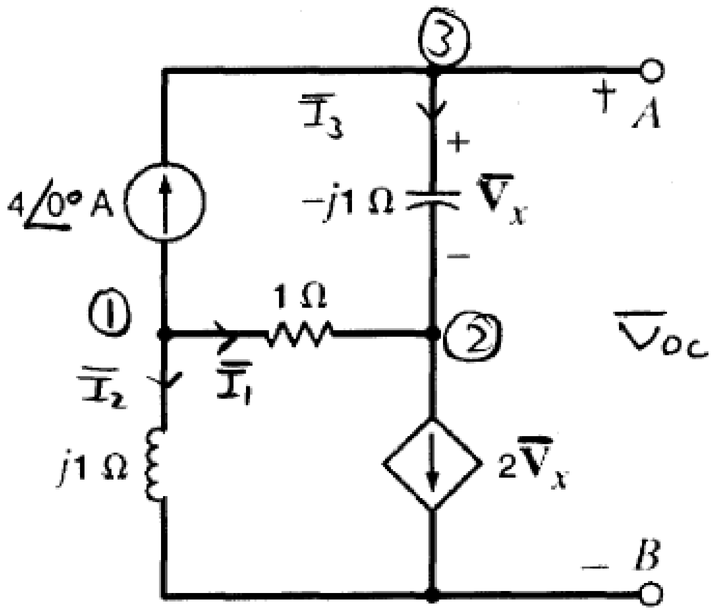


Figure P8.105

SOLUTION:



KCL at ①: $4\angle 0° + \overline{I}_1 + \overline{I}_2 = 0$

$$\frac{\overline{V}_1 - \overline{V}_2}{1} + \frac{\overline{V}_1}{j1} = -4\angle 0°$$

$$j1(\overline{V}_1 - \overline{V}_2) + \overline{V}_1 = j1(-4\angle 0°)$$

$$(1+j1)\overline{V}_1 - j1\overline{V}_2 = 4\angle -90°$$

KCL at ②: $\overline{I}_3 + \overline{I}_1 = 2\overline{V}_x$

$$\overline{V}_x = \overline{V}_{oc} - \overline{V}_2$$

$$\frac{\overline{V}_{oc} - \overline{V}_2}{-j1} + \frac{\overline{V}_1 - \overline{V}_2}{1} = 2\left[\overline{V}_{oc} - \overline{V}_2\right]$$

$$\overline{V}_{oc} - \overline{V}_2 - j1(\overline{V}_1 - \overline{V}_2) = -j2\left[\overline{V}_{oc} - \overline{V}_2\right]$$

$$-j1\overline{V}_1 + (-1-j1)\overline{V}_2 + (1+j2)\overline{V}_{oc} = 0$$

KCL at 3: $\overline{I}_3 = 4\angle 0^\circ$

$$\frac{\overline{V}_{oc} - \overline{V}_2}{-j1} = 4\angle 0^\circ$$

$$-\overline{V}_2 + V_{oc} = 4\angle -90^\circ$$

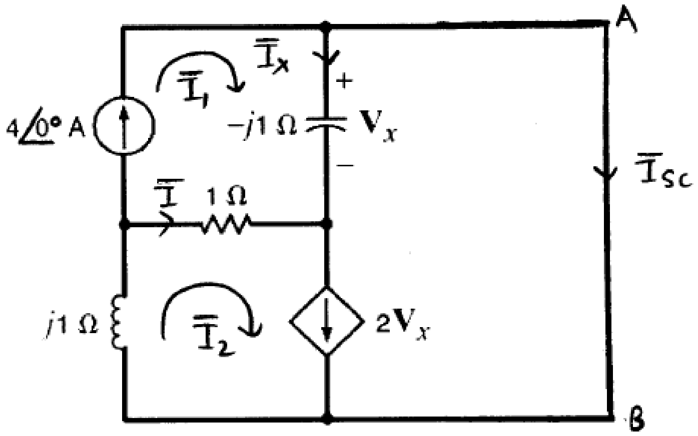
$$(1+j1)\overline{V}_1 - j1\overline{V}_2 = 4\angle -90^\circ$$

$$-j1\overline{V}_1 + (-1-j1)\overline{V}_2 + (1+j2)\overline{V}_{oc} = 0$$

$$-\overline{V}_2 + \overline{V}_{oc} = 4\angle -90^\circ$$

$$\overline{V}_1 = 8\angle 180^\circ \text{ V}$$
$$\overline{V}_2 = 8.94\ \angle 116.6^\circ \text{ V}$$

$$\overline{V}_{oc} = 5.66\ \angle 135^\circ \text{ V}$$



KCL: $\bar{I}_2 = \bar{I} + \bar{I}_1$

$$\bar{I} = \bar{I}_2 - \bar{I}_1$$

KCL: $4\angle 0° = \bar{I}_x + \bar{I}_{sc}$

$$\bar{I}_x = 4\angle 0° - \bar{I}_{sc}$$

KVL: $1(\bar{I}) - j1(-\bar{I}_x) + j1(\bar{I}_2) = 0$

$$\bar{I}_2 - \bar{I}_1 + j1(4\angle 0° - \bar{I}_{sc}) + j1\,\bar{I}_2 = 0$$

$$-\bar{I}_1 + (1+j1)\,\bar{I}_2 - j1\,\bar{I}_{sc} = 4\angle -90°$$

KCL: $2\bar{V}_x + \bar{I}_{sc} = \bar{I}_2$

$$\bar{V}_x = -j1\ (4\angle 0° - \bar{I}_{sc})$$

$$-j2(4\angle 0° - \bar{I}_{sc}) + \bar{I}_{sc} = \bar{I}_2$$

$$-\bar{I}_2 + (1+j2)\,\bar{I}_{sc} = 8\angle 90^\circ$$

$$\bar{I}_1 = 4\angle 0^\circ \text{A}$$

$$(1+j1)\,\bar{I}_2 - j1\,\bar{I}_{sc} = 5.66\angle -45^\circ$$

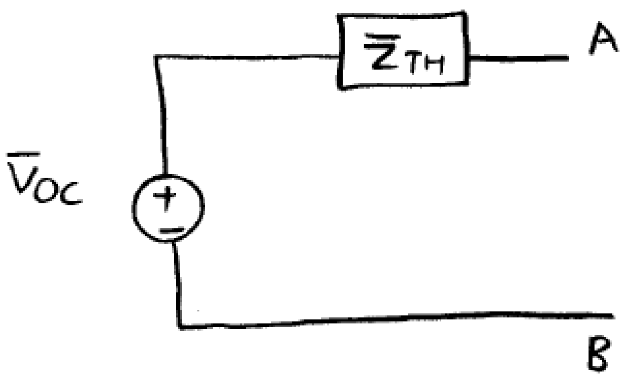
$$-\bar{I}_2 + (1+j2)\,\bar{I}_{sc} = 8\angle 90^\circ$$

$$\bar{I}_2 = 2.53\angle 71.6^\circ \text{A}$$

$$\bar{I}_{sc} = 2.53\angle 18.43^\circ \text{A}$$

$$\bar{Z}_{TH} = \frac{\bar{V}_{OC}}{\bar{I}_{sc}} = \frac{5.66\angle 135^\circ}{2.53\angle 18.43^\circ}$$

$$\bar{Z}_{TH} = 2.24\angle 116.57^\circ\,\Omega$$



8.110 Find $\mathbf{V}_o$ using Norton's theorem for the circuit in Fig. P8.110.

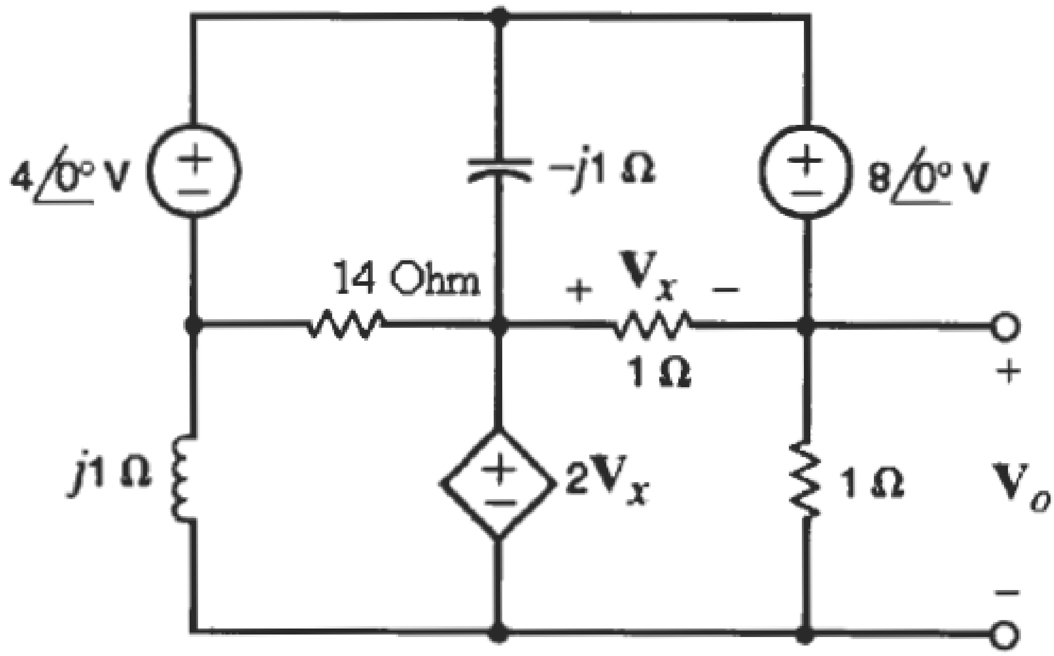
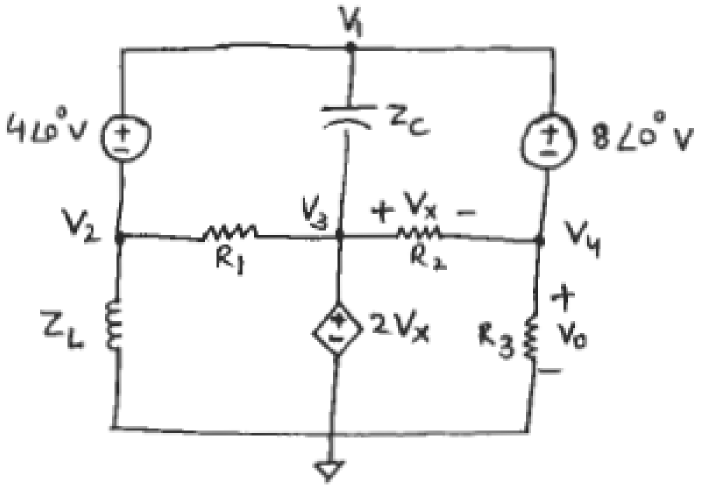


Figure P8.110

(a) Find the real part of $\mathbf{V}_o$.
(b) Find the imaginary part of $\mathbf{V}_o$.

Solution: 8-110



$R_1 = 14\,\Omega$

$R_2 = R_3 = 1\,\Omega$

$Z_C = -j1\,\Omega$

$Z_L = j1\,\Omega$

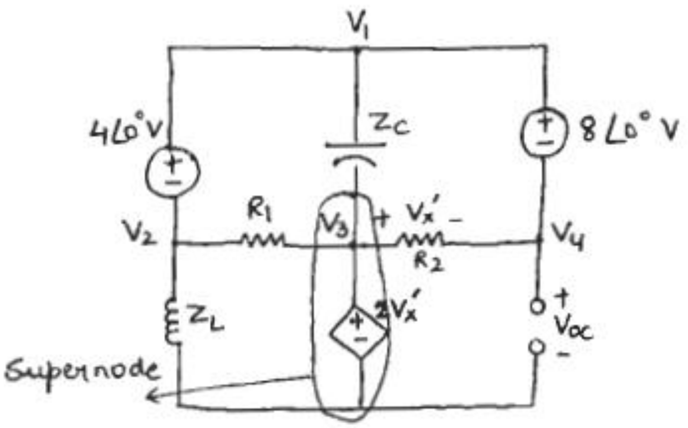
$V_1 - V_2 = 4\angle 0^\circ$ V

$V_1 - V_4 = 8\angle 0^\circ$ V

$V_1 = 8 + V_4 \quad \text{—} \quad (1)$

$V_2 = V_1 - 4$

$V_2 = V_4 + 4 \quad \text{—} \quad (2)$



$2V_x' = V_3$

$V_x' = V_3 - V_4$

$V_3 = 2(V_3 - V_4)$

$\Rightarrow V_3 = 2V_4$ —— ③

KCL @ supernode: $\frac{V_3 - V_2}{R_1} + \frac{V_3 - V_1}{Z_C} + \frac{V_3 - V_4}{R_2} - \frac{V_2}{Z_L} = 0$

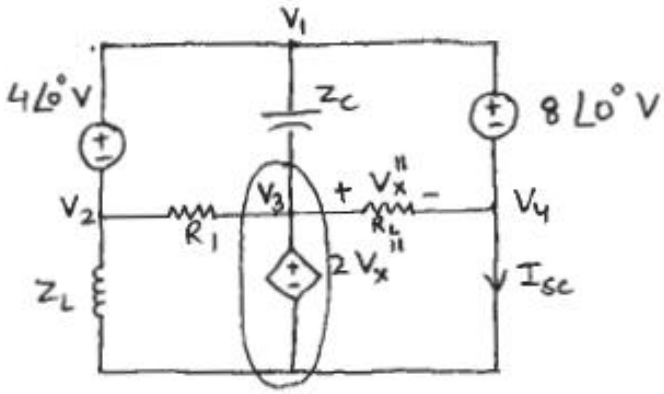
$$\frac{V_3 - V_2}{14} + \frac{V_3 - V_1}{-j1} + \frac{V_3 - V_4}{1} - \frac{V_2}{j1} = 0$$

$$V_4(15 + 28j) - (56j + 4) = 0$$

$$V_4 = 1.613 + j\,0.7215 \text{ V}$$

$$V_{oc} = V_4$$

$$\boxed{V_{oc} = 1.613 + j\,0.7215 \text{ V}}$$



$V_4 = 0$ $\quad$ $V_3 - V_4 = V_x''$ and $V_3 = 2V_x''$

$\Rightarrow V_3 = 0$

$V_1 - V_4 = 8\angle 0^\circ$

$\Rightarrow V_1 = 8 \text{ V}$

$V_1 - V_2 = 4\angle 0^\circ$

$V_2 = 4\angle 0^\circ \text{ V}$

KCL @ supernode:

$$\frac{V_3 - V_1}{Z_C} + \frac{V_3 - V_4}{R_L} + \frac{V_3 - V_2}{R_1} - \frac{V_2}{Z_L} = I_{sc}$$

$$\Rightarrow I_{sc} = -0.2857 - 4j \text{ A}$$

$$Z_{Th} = \frac{V_{oc}}{I_{sc}} = -0.208 + j\,0.388\ \Omega$$

$$V_o = I_{sc} \frac{Z_{Th} \cdot R_3}{Z_{Th} + R_3}$$

$$\Rightarrow V_o = 2.0 + j\,0.071 \text{ V}$$

(a) $\boxed{Re(V_o) = 2.0 \text{ V}}$

(b) $\boxed{Im(V_o) = -0.071 \text{ V}}$